

Indefinite Integration



$$\int (ax + b)^n = \frac{(ax+b)^{n+1}}{a(n+1)} + c, n \neq -1$$

$$\int \frac{dx}{ax + b} = \frac{1}{a} \log|ax + b| + c$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + c$$

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + c$$

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + c$$

$$\int \tan(ax + b) dx = \frac{1}{a} \log|\sec(ax + b)| + c$$

$$\int \cot(ax + b) dx = \frac{1}{a} \log|\sin(ax + b)| + c$$

$$\int \sec^2(ax + b) dx = \frac{1}{a} \tan(ax + b) + c$$

$$\int \operatorname{cosec}^2(ax + b) dx = -\frac{1}{a} \cot(ax + b) + c$$

$$\int \sec(ax + b) \cdot \tan(ax + b) dx = \frac{1}{a} \sec(ax + b) + c$$

$$\int \operatorname{cosec}(ax + b) \cdot \cot(ax + b) dx = -\frac{1}{a} \operatorname{cosec}(ax + b) + c$$

$$\int \sec x \, dx = \log|\sec x + \tan x| + c \text{ \textit{or} } \log \left| \tan \left[\frac{\pi}{4} + \frac{x}{2} \right] \right| + c$$



Integration with respect x

i) $(2x + 3)^5$

$$I = \int (2x + 3)^5 dx$$

$$= \frac{1}{2} \int t^5 dt$$

$$= \frac{1}{2} \left[\frac{t^{5+1}}{5+1} \right] + C$$

$$= \frac{1}{2} \left[\frac{t^6}{6} \right] + C = \frac{t^6}{12} + C$$

$$I = \frac{(2x+3)^6}{12} + C$$

but $2x + 3 = t$

d.w.r. to x

$$2 + 0 = \frac{dt}{dx}$$

$$2 dx = dt$$

$$dx = \frac{dt}{2}$$

ii) $\sin 2x$

$$I = \int \sin 2x \, dx$$

$$= \frac{1}{2} \int \sin t \, dt$$

$$= -\frac{1}{2} \cos t + c$$

$$I = -\frac{1}{2} \cos 2x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\text{let } 2x = t$$

$$2 = \frac{dt}{dx}$$

$$dx = \frac{dt}{2}$$

$$\text{iii)} \frac{1}{2x+1}$$

$$I = \int \frac{1}{2x+1} dx$$

$$= \int \frac{1}{t} dt$$

$$= \log|t| + c$$

$$I = \log|2x+1| + c$$

$$\text{let } 2x+1 = t$$

$$2 + 0 = \frac{dt}{dx}$$

$$dx = \frac{dt}{2}$$

iv) $\cos^3 x$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$\cos 3x + 3\cos x = 4\cos^3 x$$

$$\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$$

$$I = \int \cos^3 x \, dx$$

$$= \int \frac{\cos 3x + 3\cos x}{4} \, dx$$

$$= \frac{1}{4} \left[\int \cos 3x \, dx + 3 \int \cos x \, dx \right]$$

$$\frac{1}{4} \left[\frac{\sin 3x}{3} + 3 \sin x \right] + c$$



$$v) \frac{1}{\sqrt{x+3} - \sqrt{x+2}}$$

$$I = \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} dx$$

$$= \int \frac{1}{\sqrt{x+3} - \sqrt{x+2}} \times \frac{\sqrt{x+3} + \sqrt{x+2}}{\sqrt{x+3} + \sqrt{x+2}} dx$$

$$= \int \frac{\sqrt{x+3} + \sqrt{x+2}}{(\sqrt{x+3})^2 - (\sqrt{x+2})^2} dx = \int \frac{\sqrt{x+3} + \sqrt{x+2}}{(x+3) - (x+2)} dx$$

$$= \int (\sqrt{x+3} + \sqrt{x+2}) dx$$

$$\int (x+3)^{1/2} dx + \int (x+2)^{1/2} dx$$

$$\frac{(x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$\frac{(x+3)^{3/2}}{3/2} + \frac{(x+2)^{3/2}}{3/2} + c$$

$$\boxed{\frac{2}{3} (x+3)^{3/2} + \frac{2}{3} (x+2)^{3/2} + c}$$

vi) $x \sin x^2$

$$I = \int x \sin x^2 dx$$

$$= \frac{1}{2} \int \sin t dt$$

$$= -\frac{1}{2} \cos t + c$$

$$I = -\frac{1}{2} \cos x^2 + c$$

let $x^2 = t$

$$2x dx = dt$$

$$x dx = \frac{dt}{2}$$

vii) $\frac{x}{x^2+1}$

$$I = \int \frac{x}{x^2+1} dx$$

$$\text{let } x^2+1 = t$$

$$2x + 0 = \frac{dt}{dx}$$

$$x dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \log |t| + c$$

$$I = \frac{1}{2} \log |x^2+1| + c$$

viii) $\frac{e^x + 1}{e^x + x}$

let $e^x + x = t$

$$e^x + 1 = \frac{dt}{dx}$$

$$(e^x + 1) dx = dt$$

$$\int \frac{e^x + 1}{e^x + x} dx$$

$$\int \frac{dt}{t} = \log|t| + c = \log|e^x + x| + c$$

$$\text{ix)} \frac{e^{2x}}{e^{2x}-2}$$

$$I = \int \frac{e^{2x}}{e^{2x}-2} dx$$

$$= \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \log|t| + c$$

$$I = \frac{1}{2} \log|e^{2x}-2| + c$$

$$\text{let } e^{2x}-2=t$$

$$2e^{2x}-0 = \frac{dt}{dx}$$

$$e^{2x} dx = \frac{dt}{2}$$

$$x) \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x}$$

$$I = \int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx$$

$$= \frac{1}{2} \int \frac{dt}{t}$$

$$= \frac{1}{2} \log |t| + c$$

$$I = \frac{1}{2} \log |x^2 + \sin 2x + 2x| + c$$

$$x^2 + \sin 2x + 2x = t$$

$$2x + 2 \cos 2x + 2 = \frac{dt}{dx}$$

$$2(x + \cos 2x + 1) dx = dt$$

$$(x + \cos 2x + 1) dx = \frac{dt}{2}$$

xii) $\frac{1}{x(x^5+1)}$

$$I = \int \frac{1}{x(x^5+1)} dx$$

$$= \int \frac{x^4}{x^5(x^5+1)} dx$$

$$= \frac{1}{5} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{5} \left[\frac{1}{t} - \frac{1}{(t+1)} \right] dt$$

$$x^5 = t$$

$$5x^4 = \frac{dt}{dx}$$

$$x^4 dx = \frac{dt}{5}$$

$$\frac{1}{5} \left[\int \frac{1}{t} dt - \int \frac{1}{(t+1)} dt \right]$$

$$= \frac{1}{5} \left[\log|t| - \log|t+1| \right] + c$$

$$= \frac{1}{5} \left[\log \frac{x^5}{x^5+1} \right] + c$$

xiii) $\frac{1}{x^5(1+x^5)^{\frac{1}{5}}}$



$$\int \left(\frac{x \cos x - \sin x}{x \sin x} \right) dx$$

$$I = \int \frac{x \cos x - \sin x}{x \sin x} dx$$

$$= \int \frac{x \cos x + \sin x - \sin x - \sin x}{x \sin x} dx$$

$$= \int \left[\frac{x \cos x + \sin x}{x \sin x} - \frac{2 \sin x}{x \sin x} \right] dx$$

$$= \int \frac{1}{t} dt - \int \frac{2}{x} dx = \log t - 2 \log x + c$$

$$= \log(x \sin x) - \log x^2 + c$$

$$x \sin x = t$$

$$x \cos x + \sin x = \frac{dt}{dx}$$

$$(x \cos x + \sin x) dx = dt$$

$$\log \frac{x \sin x}{x^2} + c$$

$$\log \left| \frac{\sin x}{x} \right| + c$$

$$2x^3 e^{x^2}$$

